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SPLITTING OF RESONANT FREQUENCIES OF ACOUSTIC WAVES IN ROTATING COMPRESSIBLE FLUID

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It is shown that in a rotating compressible fluid the resonant frequencies (measured in a system of reference rotating together with the medium) for the azimuthally running acoustic waves are split into two components. The received results can be of practical significance as a basis of a method of measurements of angular speed of medium and for acoustics of rotating technical devices.

As is known [1], in a compressible fluid, localized between two rigid coaxial cylindrical walls, free oscillations of the character of both standing and running in an azimuthal direction waves are possible. With the help of the equations

$$\rho_0 \dot{\vec{V}} = -\vec{\nabla}\tilde{p}\,, \quad \dot{\tilde{p}} = -\rho_0 c_0^2 \, div\, \vec{V} \tag{1}$$

it is easy to show that for the standing waves, in the absence of axial component of oscillation speed $\vec{V}$ (i. e., for $V_z = 0$), the acoustic pressure $\tilde{p} = p - p_0$ looks like $\tilde{p}(r,\varphi,t) = P_m(r)\cos m\,\varphi \ \ \cos\omega_{mn}t$, and for the azimuthally running waves $\tilde{p}(r,\varphi,t) = P_m(r)\cos(m\,\varphi - \omega_{mn}t)$. Here, $P_m(r) = A \ \ J_m(\omega_{mn}r/c_0) + B \ \ Y_m(\omega_{mn}r/c_0)$, A and B are the constants, $r, \varphi, \ z$ are the cylindrical coordinates, J_m and Y_m are the Bessel functions [2], $m \ \ n$ are the integers, p_0, ρ_0 and c_0 are the equilibrium pressure, density and sound speed. The sets of resonant frequencies ω_{mn} for both cases are identical and can be found from the equation

$$J_m'\left(\omega R_1/c_0\right) Y_m'\left(\omega R_2/c_0\right) = J_m'\left(\omega R_2/c_0\right) Y_m'\left(\omega R_1/c_0\right), \tag{2}$$

where R_1 and R_2 are the radiuses of the walls, $J_m'(x) = dJ_m/dx$, $Y_m'(x) = dY_m/dx$.

Let us consider the behaviour of such waves on the background of the equilibrium state of the medium rotating with a constant angular speed $\vec{\Omega}$. Nondissipative hydrodynamic processes in the rotating compressed liquid (gas), considered from the point of view of the system of reference S rotating together with the fluid, are described by system of equations

$$\begin{aligned} &\dot{\vec{V}} + (\vec{V}\cdot\vec{\nabla})\vec{V} = -\,\rho^{-1}\vec{\nabla}p + \Omega^2\,\vec{r} + 2\,[\vec{V}\times\vec{\Omega}], \\ &\dot{\rho} + div(\rho\vec{V}) = 0, \qquad \dot{p} + \vec{V}\cdot\vec{\nabla}p = c^2(\dot{\rho} + \vec{V}\cdot\vec{\nabla}\rho), \end{aligned} \tag{3}$$

where $\vec{V}$ is the velocity of a medium element relative to S. Within the limits of linear acoustics and for relatively low speeds of rotation (i. e., for small value of $M = \Omega R/c$), system (3), in view of a condition of equilibrium $\vec{\nabla}p_0 = \rho_0\Omega^2\,\vec{r}$, leads (instead of (1)) to the following equations:

$$\dot{\vec{V}} = -\rho_0^{-1}\vec{\nabla}\tilde{p} + 2[\vec{V}\times\vec{\Omega}], \tag{4}$$

$$\dot{\tilde{p}} = -\rho_0 c_0^2\; div\,\vec{V}. \tag{5}$$

System (4), (5) describes adequately the corrections of the first order in M, caused by the influence of the medium rotation on oscillations, if the conditions $M \ll 1$ and $V_A/c \sim M^2$ are satisfied (V_A is the amplitude of the velocity of oscillations). Thus the ratio Ω/ω for the resonant values of ω will be of the order M, and p_0, ρ_0, and c_0 in equations (4), (5) can be considered constant.

This allows to use system (4), (5), for example, for the frequencies of rotation of the order 1 – 10 r.p.s. and the radial sizes 0.2 – 0.5 m for the sound intensity corresponding to the values $V_A/c < 10^{-5}$ for both gases and liquids.

It is easy to check that, in the approximation considered, elimination of $\vec{V}$ from (4), (5) leads to the same differential equation for the acoustic pressure $\tilde{p}$ as in the case of non-rotating medium:

$$\ddot{\tilde{p}} = c_0^2\,\nabla^2\,\tilde{p}, \quad \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}. \tag{6}$$

At the same time, the boundary condition for $\tilde{p}(r,\varphi,t)$ on a rigid cylindrical wall of the radius R (that follows from the requirement $V_r(r = R) = 0$) gets, by virtue of (4), an additional term, which have the first order in Ω/ω:

$$\left(\frac{\partial\dot{\tilde{p}}}{\partial r} + \frac{2\Omega}{r}\frac{\partial\tilde{p}}{\partial\varphi}\right)_{r=R} = 0. \tag{7}$$

As for the azimuthally running wave (i. e., for the harmonic of number $m = \pm 1,\ \pm 2,\ \pm 3\ldots)$ the acoustic pressure $\tilde{p}(r, \varphi, t)$ looks like $P_m(r)\cos(m\,\varphi - \omega t)$, it follows from condition (7):

$$\left(\frac{dP_m}{dr} - \frac{2\Omega m}{\omega r}P_m\right)_{r=R_{\,1,\,2}} = 0, \tag{8}$$

where R_1 and R_2 are the radiuses of rigid coaxial cylindrical walls between which the wave is localized.

According to equation (6), when $\Omega \neq 0$, $P_m(r)$ holds the form $A\ J_m(\omega r/c_0) + B\ Y_m(\omega r/c_0)$. Substituting $P_m(r)$ in (8), we receive, instead of (2), the condition

$$\frac{J'_m\left(\dfrac{\omega R_1}{c_0}\right) - \dfrac{2mc_0\Omega}{\omega^2 R_1}J_m\left(\dfrac{\omega R_1}{c_0}\right)}{J'_m\left(\dfrac{\omega R_2}{c_0}\right) - \dfrac{2mc_0\Omega}{\omega^2 R_2}J_m\left(\dfrac{\omega R_2}{c_0}\right)} = \frac{Y'_m\left(\dfrac{\omega R_1}{c_0}\right) - \dfrac{2mc_0\Omega}{\omega^2 R_1}Y_m\left(\dfrac{\omega R_1}{c_0}\right)}{Y'_m\left(\dfrac{\omega R_2}{c_0}\right) - \dfrac{2mc_0\Omega}{\omega^2 R_2}Y_m\left(\dfrac{\omega R_2}{c_0}\right)}. \tag{9}$$

Values of ω satisfying (9) form a spectrum of the resonant frequencies of the azimuthally running waves: ω_{mn} at $\Omega = 0$, ω^+_{mn}, if $m\,\Omega > 0$ (i. e., for a wave running in a direction of medium rotation) and ω^-_{mn}, if $m\,\Omega < 0$.

Frequencies $\nu_{mn} = 2\pi\omega_{mn}$ and frequency deviations $\nu^+_{mn} - \nu_{mn}$ and $\nu^-_{mn} - \nu_{mn}$, where $\nu^\pm_{mn} = 2\pi\omega^\pm_{mn}$ obtained by a numerical solution of equation (9) at $R_1 = 0.2$ m, $R_2 = 0.5$ m, $\Omega = \pm 2\pi N$, $N = 10$ r.p.s, and $c_0 = 330$ m/s (for m= 1, 2, 3; n=1, 2, 3, 4, 5) are presented in Table 1.

m\n	1	2	3	4	5
1	153.55 -3.91 1.32	594.45 1.41 - 1.44	1122.19 0.43 - 0.43	1664.72 0.20 - 0.20	2211.01 0.11 - 0.11
2	298.57 - 5.24 4.06	673.95 2.29 - 2.39	1161.34 0.86 - 0.87	1690.30 0.40 - 0.40	2230.04 0.23 - 0.23
3	431.53 -6.12 5.43	791.52 2.25 - 2.40	1225.43 1.27 - 1.29	1732.52 0.60 - 0.61	2261.55 0.34 - 0.34

Table 1: Frequencies ν_{mn}(Hz) and deviations: $\nu^+_{mn} - \nu_{mn}$, $\nu^-_{mn} - \nu_{mn}$ (Hz).

For a cylinder of the radius R (i. e., for R_1= 0, $R_2 = R$) the function $P_m(r)$ is reduced to $A\ J_m(\omega\, r/c_0)$, and we can receive the approximated expression for the resonant frequencies in an explicit form:

$$\omega_{mn} + \Delta\omega_{mn} = \frac{c_0}{R} X_{mn} + \frac{2\, m\, \Omega\, J_m\,(X_{mn})}{X_{mn}^2\; J_m''\,(X_{mn})}, \tag{10}$$

where X_{mn} are the roots of the equation $J_m'(x) = 0$.

According to (10), ratio of splitting of frequencies to angular speed of medium in this case is of a universal character:

$$f_{mn} = \frac{\omega_{mn}^+ - \omega_{mn}^-}{\Omega} = \frac{4\, m\; J_m\,(X_{mn})}{X_{mn}^2\; J_m''\,(X_{mn})}. \tag{11}$$

Table 2 gives the values of the roots X_{mn} and of the "splitting coefficients" f_{mn}.

m\n	1	2	3	4	5
1	1.841 - 1.674	5.331 - 0.146	8.536 - 0.056	11.706 - 0.029	14.864 - 0.018
2	3.054 - 1.501	6.706 - 0.195	9.969 - 0.084	13.170 - 0.047	16.348 - 0.030
3	4.201 - 1.387	8.015 - 0.217	11.346 - 0.100	14.586 - 0.059	17.789 - 0.039

Table 2: Values of X_{mn} and f_{mn} for m = 1, 2, 3; n = 1, 2, 3, 4, 5.

As the frequencies of the running waves following and opposing a direction of rotation are not equal, it is impossible to construct a standing wave as some their superposition. The absence of standing waves for $m\,\Omega \neq 0$ follows directly from impossibility to satisfy boundary conditions (7) by the pressure $\tilde{p}(r, \varphi, t)$ of the form $P_m(r) \cos m\,\varphi\;\; \cos \omega t$. For $m\;\Omega \neq 0$, a pulse of waves moving in an azimuthal direction with the angular speed Ω_{mn}^{puls}, which is equal to $(\omega_{mn}^+ - \omega_{mn}^-)/(2m)$ relative to rotating medium, will occur instead of a standing wave.

The splitting of sound frequencies in a rotating medium can be, in some sense, considered as an acoustic analog of the Sagnac effect in optics [3].

For values of parameters $M = \Omega R/c$ and V_A/c, which are rather greater than considered above, taking into account of nonlinear members in equations and corresponding correction of the received results are necessary. We expect, however, that, at least for weak nonlinearity, the effect of splitting of frequencies will be of similar character, and that the resonant response of medium will begin to be also revealed on the frequencies of pulses.

References

[1] Isakovich M. A. General Acoustic. Moscow. 1973. P. 271.

[2] Handbook of mathematical functions. Edited by M. Abramovitz, I. Stegun, NBSAMS, 1964.

[3] Landau L. D., Lifshits E. M. Theory of Field. Moscow. 1988. P. 326.